

Magnetic Catalysis vs Magnetic Inhibition

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We discuss the fate of chiral symmetry in an extremely strong magnetic field B . We investigate not only quark fluctuations but also neutral meson effects. The former would enhance the chiral-symmetry breaking at finite B according to the Magnetic Catalysis, while the latter would suppress the chiral condensate once B exceeds the scale of the hadron structure. Using a chiral model we demonstrate how neutral mesons are subject to the dimensional reduction and the low dimensionality favors the chiral-symmetric phase. We point out that this effect, the Magnetic Inhibition, can be a feasible explanation for recent lattice-QCD data indicating the decreasing behavior of the chiral-restoration temperature with increasing B .

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Theorists have been pursuing the answer of questions in various extremes. What happens if the temperature is extremely high? According to the theory of the strong interaction, namely, quantum chromodynamics (QCD), chiral symmetry should be restored and color degrees of freedom should be released at the temperature T of the order of $\Lambda_{\text{QCD}} \sim 200$ MeV [1–3]. If T is raised further, the electroweak phase transition should take place at $T \sim 100$ GeV. Along the same spirit many theorists are trying to clarify what could happen at extremely high baryon density. In the QCD asymptotic regime where the perturbative calculation should work, theoretical considerations predict the color superconducting phases [2, 4, 5]. In particular the ground state should form the color-flavor locking (CFL) if the quark chemical potential, μ_q , is sufficiently larger than the strange quark mass, while there may appear many other pairing patterns such as the uSC phase, the dSC phase, the 2SC phase, etc in the intermediate density region. It is still a big theoretical challenge to identify the correct phase structure of QCD on the whole μ_q - T plane.

Recently the QCD phase structure in the presence of strong magnetic field B has been revisited extensively for several good reasons [3]: First, QCD matter under strong B is worth thinking as a realistic situation in non-central collisions in the relativistic heavy-ion experiment as conducted at the Relativistic Heavy-Ion Collider (RHIC) and the Large Hadron Collider (LHC). A simple calculation gives us an order estimate for the produced magnetic field as $eB > \Lambda_{\text{QCD}}^2$ at the RHIC energy [6]. Second, the B -effect is very similar to the baryon chemical potential in a sense that gluons have no direct coupling and the QCD equation of state is affected only through quark polarization processes. The Monte-Carlo simulation with B is, in spite of the similarity to μ_q , possible without the notorious sign problem [7, 8], which is a great theoretical advantage. Third, it is certainly intriguing to address such a simple and well-defined question; what is the ground

state of QCD matter when a very strong magnetic field, $B \gg \Lambda_{\text{QCD}}$, is applied?

It should be tough in general to answer to such a question since the confinement/deconfinement phenomena belong to the gluon dynamics and B -effects are then indirect. (See Ref. [9] for recent attempts.) This difficulty is common also in the finite- μ_q analysis, which hinders the QCD phase diagram research also [10]. In other words, one could never reach the correct QCD phase diagram on the μ_q - T plane until one can establish a machinery to reveal the B -effects that should be under better theoretical control [11]. In a related context the new entanglement with finite B and μ_q is also an interesting research subject [12].

In contrast to the confinement sector, the properties of chiral symmetry reside in the quark part, and thus they are directly sensitive to the presence of B . It would be, therefore, more tractable to focus on chiral symmetry in the strong B limit. From this point of view of the interplay between chiral symmetry and B , the *Magnetic Catalysis* is one of the most significant phenomena [13–15]: In chiral quark models such as the Nambu–Jona-Lasinio (NJL) model without the confinement effect, chiral symmetry is spontaneously broken for a sufficiently strong coupling constant, i.e. $G > G_c$. With the magnetic field, on the other hand, a non-zero value of the chiral condensate would be an inevitable consequence from the Landau zero-mode contribution regardless of the value of G . Fermions are always massive in the presence of B , hence, even though they are massless at the Lagrangian level (i.e. the chiral limit). In fact, the (3+1) dimensional NJL model with $U(1)_L \times U(1)_R$ chiral symmetry [14] [this is the unbroken part of chiral symmetry with B that breaks isospin symmetry explicitly] is defined by the Lagrangian,

$$\mathcal{L} = \bar{\psi} i \not{D} \psi + \frac{G}{2} [(\bar{\psi}\psi)^2 + (\bar{\psi} i \gamma_5 \psi)^2], \quad (1)$$

where $\mathcal{D} \equiv \gamma^\mu (\partial_\mu + ieA_\mu)$ with $A^\mu = (0, -yB/2, xB/2, 0)$ (in the symmetric gauge) and $B > 0$. Then it has been established that the constituent quark mass is expressed analogously to the gap obtained in the BCS theory;

$$m^2 = \frac{eB}{\pi} \exp\left(-\frac{4\pi^2}{eBG}\right), \quad (2)$$

which is non-zero for any G . One can understand Eq. (2) in such a way that B is a catalyst to induce a non-vanishing chiral condensate, that is, the Magnetic Catalysis. In the NJL model as an effective description of QCD at low energy, G has the energy scale comparable to Λ_{QCD} , i.e. $G \sim \Lambda_{\text{QCD}}^{-2}$. This means, together with Eq. (2), that the B -induced value of m^2 becomes appreciable for $eB \sim \Lambda_{\text{QCD}}^2$.

It is natural, as suggested by the Magnetic Catalysis, that B should enhance the chiral-symmetry breaking, so that the critical temperature, T_c , for chiral restoration should increase with increasing B , which is indeed the case in all chiral-model calculations (see Ref. [16] for a recent review). The latest lattice-QCD data, however, supports dropping behavior of T_c as a function of B and there is no clear physical explanation for this. The main goal of the present work is to propose a new mechanism for chiral restoration at strong B that could be a feasible explanation for the lattice-QCD data.

Our idea is as follows. Because $U(1)_L \times U(1)_R$ is spontaneously broken down to $U(1)_V$, the Nambu-Goldstone (NG) boson (i.e. π^0) must exist as a composite of fermions. It does not matter whether π^0 is a tight bound-state particle of QCD or not, but here let us just call this NG boson π^0 in our QCD-based convention. If B is extremely strong, fermions that form π^0 are affected by B and eventually their motions are restricted along the B -direction. Hence, it should be conceivable that π^0 also undergoes the dimensional reduction to the (1+1)-dimensional dynamics. Once this happens, the spontaneous chiral-symmetry breaking is prohibited according to Marmin-Wagner's theorem [17]. Such a possibility, that we name the *Magnetic Inhibition*, was considered partially in Ref. [14], but only the approximated form of the π^0 propagator was discussed there. In this work we will fully evaluate the π^0 propagator to address its dynamical change in strong B and formulate the above-mentioned idea.

For later convenience let us look closely at the derivation of Eq. (2) using the quark propagator on top of the vector potential A^μ . The quark propagator is expressed as $S(p) = \sum_{n=0}^{\infty} iS_n(p)/(p_{\parallel}^2 - m_n^2)$ with $m_n^2 \equiv m^2 + 2eBn$ and $S_n(p) \equiv (\not{p}_{\parallel} + m)[P_+ A_n(p_{\perp}^2) + P_- A_{n-1}(p_{\perp}^2)] + \not{p}_{\perp} B_n(p_{\perp}^2)$. Here we have introduced several notations: $p_{\parallel}^2 \equiv p_0^2 - p_z^2$, $p_{\perp}^2 \equiv p_x^2 + p_y^2$, $A_n(p_{\perp}^2) \equiv 2e^{-2z}(-1)^n L_n^{(0)}(4z)$, and $B_n(p_{\perp}^2) \equiv 4e^{-2z}(-1)^n L_{n-1}^{(1)}(4z)$ with $z \equiv p_{\perp}^2/(2eB)$ and the projection operators; $P_{\pm} \equiv (1 \pm i\gamma^1\gamma^2)/2$. The generalized

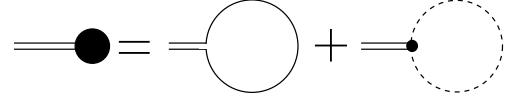


FIG. 1. Schematic expression of the gap equation. The solid curve represents the quark loop with the quark propagator $S(p)$ and the four-Fermi coupling G and the dashed curve represents the pion loop with the pion propagator D_π and the coupling $\Gamma_{\sigma\pi\pi}$.

Laguerre polynomials are defined as usual by $L_n^{(\alpha)}(x) \equiv (e^x x^{-\alpha}/n!)(d^n/dx^n)(e^{-x} x^{n+\alpha})$ [14, 18].

The most important ingredient to study the Magnetic Catalysis is the gap equation at the quark one-loop level,

$$0 = \frac{m}{G} - \int \frac{d^4 p}{(2\pi)^4} \text{tr} S(p) = \frac{m}{G} - \frac{m}{2\pi} \cdot \frac{eB}{2\pi} \int_{1/\Lambda^2} \frac{ds}{s} e^{-m^2 s} \coth(eBs), \quad (3)$$

which is regularized in the proper-time method.

This represents the tadpole diagram in terms of quarks (see the loop diagram with a solid line in Fig. 1). Obviously $m = 0$ is a solution of the above equation. We have to make a comparison in energy to locate the ground state that should have the lowest energy. It is a straightforward exercise to evaluate the effective potential by integrating the gap equation with respect to m , which leads to $V(m) = m^2/(2G) + V_q(m)$, where the quark part is

$$V_q(m) = \frac{eB}{8\pi^2} \int_{1/\Lambda^2} \frac{ds}{s^2} e^{-m^2 s} \coth(eBs) + (\text{const.}). \quad (4)$$

The onset for the spontaneous chiral-symmetry breaking in the chiral limit is immediately located from the sign of the potential curvature, i.e. the coefficient of the m^2 -term in the potential. In the $B = 0$ case, we can expand as $V_q(m) \simeq -(\Lambda^2/8\pi^2)m^2 + (\text{const.})$, from which we can conclude that chiral symmetry is spontaneously broken only for $G\Lambda^2 > 4\pi^2$. [Note that we did not consider the color factor here.] In the limit of $eB \gg \Lambda^2$, on the other hand, only the lowest Landau level (Landau zero-mode) can contribute to the gap equation, or, we can approximate the gap equation as $\coth(eBs) \simeq 1$ to find

$$V_q(m) \simeq \frac{eB}{8\pi^2} \left(\Lambda^2 - m^2 \ln \frac{e^{1-\gamma}\Lambda^2}{m^2} + \mathcal{O}(m^3) \right). \quad (5)$$

We see that the potential curvature has a logarithmic singularity at $m = 0$ and thus the curvature can be always negative for sufficiently small m , which means that the symmetric state with $m = 0$ cannot be realized. This is how the Magnetic Catalysis works. The extremal point of $m^2/(2G) + V_q(m)$ with Eq. (5) gives a gap equation whose solution reads $m^2 = e^{-\gamma}\Lambda^2 e^{-4\pi^2/(eBG)}$. We note that the difference in the overall coefficient from Eq. (2) originates from whether $eB \gg \Lambda^2$ or not. As long as

$eB \ll \Lambda^2$, the dynamical quark mass is characterized by eB , but once eB exceeds the order of Λ^2 , the quark mass squared is no longer proportional to eB but suppressed by another (smaller) scale of Λ^2 .

Now let us proceed to the calculations including the pion-loop effects under strong B . Because charged pions are as massive as eB , we can simply discard π^\pm and focus only on π^0 , which justifies the usage of our simple model setting with only $U(1)_L \times U(1)_R$ except for the color factor. As long as eB is small as compared to the pion size, we can treat π^0 as a point particle as in the chiral perturbation theory [15, 19]. However, π^0 is a composite particle, and it is conceivable that the dispersion relation of π^0 should be significantly modified by B . We can concretely investigate this by constructing π^0 dynamically in the present model. In the conventional random phase approximation the pion propagator is

$$iD_\pi^{-1}(p) = -\frac{1}{G} + i \int \frac{d^4k}{(2\pi)^4} \text{tr}[\gamma_5 S(k) \gamma_5 S(p+k)] . \quad (6)$$

After some (tedious) calculations we can find the following expression:

$$iD_\pi^{-1} = -\hat{m}_\pi^2 + \frac{eB}{2\pi} e^{-z} \sum_{n,l=0}^{\infty} \frac{l!}{n!} z^{n-l} i\Pi_2(p_\parallel^2, m_n^2, m_l^2) \times [p_\parallel^2 F_{nl}^\parallel(z) - p_\perp^2 F_{nl}^\perp(z)] , \quad (7)$$

where we introduced new notations to indicate some combinations of the Laguerre polynomials, $F_{nl}^\parallel(z) \equiv [L_l^{(n-l)}(z)]^2 + (n/l)[L_{l-1}^{(n-l)}(z)]^2$ and $F_{nl}^\perp(z) \equiv (z/l)[L_{l-1}^{(n-l+1)}(z)]^2 + (n/z)[L_l^{(n-l-1)}(z)]^2$. Also, we defined

$$\Pi_2(p_\parallel^2, m_n^2, m_l^2) \equiv \int \frac{d^2k_\perp}{(2\pi)^2} \frac{i}{k_\parallel^2 - m_n^2} \frac{i}{(k_\parallel + p_\parallel)^2 - m_l^2} , \quad (8)$$

with $m_n^2 = m^2 + 2eBn$ and $m_l^2 = m^2 + 2eBl$. The expression looks complicated and it would be convenient to approximate it as

$$iD_\pi^{-1} \approx Z_\pi^{-1} (p_\parallel^2 - v_\perp^2 p_\perp^2 - m_\pi^2) , \quad (9)$$

that is motivated from an expansion valid for $p_\perp^2 < eB$ and $p_\parallel^2 < m^2$. The explicit computation gives $Z_\pi^{-1} = (1/8\pi^2)[eB/m^2 + \ln(e^{-\gamma}\Lambda^2/2eB) - \psi(1 + m^2/2eB)]$ and $v_\perp^2 = (Z_\pi/8\pi^2) \ln(e^{-\gamma}\Lambda^2/m^2)$ with $\psi(x)$ being the digamma function [14]. We note that the physical pion mass is given by $m_\pi^2 \equiv Z_\pi \hat{m}_\pi^2$ using the bare pion mass \hat{m}_π in Eq. (7).

To see how Eq. (9) works, we numerically evaluate the full propagator (7) to make a plot for the dispersion relation in Fig. 2. As long as Eq. (9) is a sensible approximation of Eq. (7), the zero of the pion propagator inverse should behave like $p_\parallel^2 = v_\perp^2 p_\perp^2 + m_\pi^2$, which is clearly confirmed in Fig. 2. Furthermore, this type of the dispersion form persists even for $p_\perp^2 > eB$ and/or $p_\parallel^2 > m^2$.

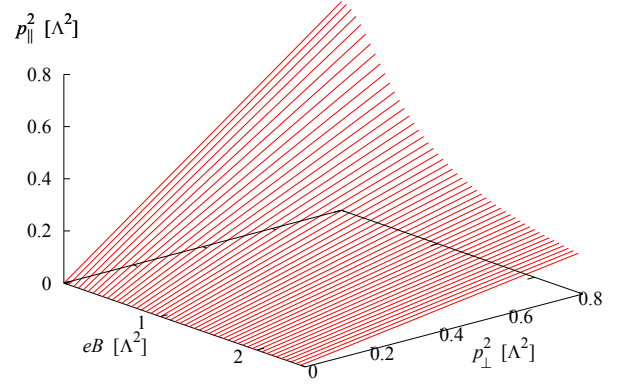


FIG. 2. Zero of the pion propagator inverse (7) as a function of p_\parallel^2 , p_\perp^2 , and eB in the unit of Λ^2 . We chose $m = 0.5\Lambda$ to avoid unphysical threshold effects. The slope of p_\parallel^2 against p_\perp^2 corresponds to the transverse velocity v_\perp^2 . Clearly the expanded form (9) is a good approximation even for $p_\perp^2 > eB$ and/or $p_\parallel^2 > m^2$.

Concerning the properties of Z_π and v_\perp^2 , here, the essential point is that $Z_\pi^{-1} \sim \mathcal{O}(eB)$ and thus Z_π goes smaller with increasing eB . This makes the transverse velocity of π^0 behave as $v_\perp^2 \sim Z_\pi \sim 1/(eB)$ that goes smaller accordingly. Such vanishing behavior of v_\perp^2 is nothing but the concrete realization of the dimensional reduction from the (3+1)- to the (1+1)-dimensional dynamics.

We would make a remark about the nature of the dimensional reduction for quarks and pions. One might have thought, at a first glance, that the dimensional reduction with $v_\perp^2 \rightarrow 0$ in Eq. (9) seems to be a different situation from quarks under strong B . In an intuitive picture quarks are trapped by B and the transverse motion is highly restricted, so that quarks can move only along B , which is how the dimensional reduction occurs for quarks. On the other hand, the neutral pion costs no energy to move and thus travels freely in the transverse directions when $v_\perp^2 = 0$. Such intuitive descriptions may sound far different but the underlying physics is common. Actually the Landau level for quarks is just a quantum number, and even for the Landau zero-mode for example, the p_\perp -integration should be carried out, which picks up the Landau degeneracy factor, $eB/(2\pi)$. In the same manner the p_\perp -integration for pions should count the density of states.

We shall explicitly go into the computation of the pion loop as depicted by the dashed line in Fig. 1. For $p_\parallel^2 > 4m^2$ the full propagator given by Eq. (7) would suffer from the threshold effect associated with the $\pi^0 \rightarrow q\bar{q}$ decay which is unphysical due to the lack of confinement. We can evade this artifact by keeping the approximate form (9), the validity of which is checked in Fig. 2. We also cut off the transverse momentum integration in the range $p_\perp^2 \lesssim eB$, which is reminiscent of the coefficient in

Eq. (2); the transverse degeneracy factor should be either eB or Λ^2 that is smaller than the other. The microscopic origin of this cutoff by eB is the p_\perp -dependence in v_\perp^2 . Not to rely on model-dependent details, we postulate the form (9) and introduce a sharp cutoff with an unknown parameter ξ as

$$\begin{aligned} & \frac{1}{2} \int_{p_\perp^2 < \xi eB} \frac{d^4 p}{(2\pi)^4} \Gamma_{\sigma\pi\pi}(p) D_\pi(p) \\ &= 2m \int_{p_\perp^2 < \xi eB} \frac{d^4 p}{(2\pi)^4} \frac{i}{p_\parallel^2 - v_\perp^2 p_\perp^2 - m_\pi^2} \\ &= 2m \int_{1/\Lambda_\pi^2}^\infty ds \int_{p_\perp^2 < \xi eB} \frac{d^4 \tilde{p}}{(2\pi)^4} e^{-s(\tilde{p}_\parallel^2 + v_\perp^2 p_\perp^2 + m_\pi^2)}, \quad (10) \end{aligned}$$

where the triple meson vertex is given by $\Gamma_{\sigma\pi\pi}(p) = -(\delta/\delta m)iD_\pi^{-1}(p)$ that we approximate at vanishing momentum by $\Gamma_{\sigma\pi\pi}(0) = 4m/Z_\pi$. We have used the Wick rotation from p to Euclidean \tilde{p} and implemented the proper-time regularization again with the UV cutoff Λ_π that is in principle related to the cutoff Λ in the quark sector. This p -integration is finite and results in the following expression;

$$\begin{aligned} & \frac{m}{8\pi^2 v_\perp^2} \int_{1/\Lambda_\pi^2}^\infty \frac{ds}{s^2} e^{-sm_\pi^2} (1 - e^{-s\xi eB v_\perp^2}) \\ & \simeq \frac{m}{8\pi^2} \xi eB \ln \frac{\Lambda_\pi^2 e^{1-\gamma}}{eB v_\perp^2} + \mathcal{O}(m_\pi^2). \quad (11) \end{aligned}$$

When the magnetic field is extremely strong, the wavefunction renormalization behaves as $Z_\pi = 8\pi^2 m^2/(eB)$ in the leading-order of eB , and the velocity is $v_\perp^2 = m^2/(eB) \ln(e^{-\gamma} \Lambda^2/m^2)$ accordingly. The contribution to the potential energy then becomes

$$\begin{aligned} V_\pi(m) &= \int^m dm' \frac{m'}{8\pi^2} \xi eB \ln \frac{\Lambda_\pi^2 e^{1-\gamma} \ln(e^{-\gamma} \Lambda^2/m'^2)}{m'^2} \\ &\simeq \xi \frac{eB m^2}{16\pi^2} \ln \left[\frac{e^{2-\gamma} \Lambda_\pi^2}{m^2} \right] + (\text{const.}), \quad (12) \end{aligned}$$

where we keep only the dominant term for small m and drop negligible terms $\propto m^2 \ln(\ln m^2)$ and $m^2/\ln m^2$. The potential contribution from the pion loops has a singularity at $m = 0$ and leads to a divergingly positive curvature at small m , which favors chiral-symmetric phase in a way opposite to the quark potential in Eq. (5). Since $V_\pi(m)$ encompasses an opposite effect to the Magnetic Catalysis, we would call this the Magnetic Inhibition. Interestingly the logarithmic singularity associated with the Magnetic Inhibition is of the same strength as that with the Magnetic Catalysis and these two effects should compete at sufficiently strong B . Our main purpose in this work is to propose a new physical mechanism, the Magnetic Inhibition, leading to a singularity $\sim m^2 \ln m^2$, and the determination of the singularity coefficient would require more works (and possibly depend on details of model assumptions).

Here we would emphasize that our results are qualitatively consistent with the latest lattice-QCD data indicating the decreasing behavior of chiral T_c with increasing B . At $T \neq 0$ the logarithmic singularity in Eq. (5) responsible for the Magnetic Catalysis vanishes due to the absence of the Matsubara zero-mode for fermions and the Magnetic Catalysis is significantly weakened [20]. The Magnetic Inhibition is, on the other hand, enhanced by the temperature effects since the Matsubara zero-mode for bosons should be accompanied by a stronger infrared singularity. Therefore, the Magnetic Inhibition can soon overcome the Magnetic Catalysis at finite temperature. This could be a feasible explanation for the lattice-QCD data [8]. In fact, the lattice-QCD data implies that, for a fixed value of T , chiral symmetry is restored as a function of increasing eB . Quantitative analyses on this phase transition induced by the Magnetic Inhibition should deserve more investigations including lattice-QCD simulations and chiral-model approaches.

Finally, we would stress that the present work is the first attempt to exemplify the importance of the hadron structural change in a strong magnetic field. Our analysis could be extended to investigate, for example, the possibility of the meson condensation induced by the B -effect [21].

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